Equation of curve with origin at left support and xx axis through supports:

\[ y = \frac{4dx}{s} (s-x) \quad \text{--- (1)} \]

or \[ y = \frac{wx(s-x)}{2H} \quad \text{--- (1a)} \]

Horizontal cable tension, \( H = \frac{w1^3}{2d} \text{ or } \frac{ws^3}{8d} \) \quad \text{--- (2)}

\[ \tan z = \frac{2d}{l} \frac{4d}{s} \quad \text{--- (3)} \]

\[ \sec z = \sqrt{1 + \frac{16d^2}{s^2}} \quad \text{--- (4)} \]

\[ T = H \cdot \sec z = H \sqrt{1 + \frac{16d^2}{s^2}} \quad \text{--- (5)} \]

\[ M = d - y = \frac{a^2d}{F} \quad \text{--- (6)} \]

Suspended length between supports, \( L \):

\[ L = s + \frac{8d^3}{3s} \text{ where } "d" \text{ is 5% of } "s" \text{ or less (approx.)} \text{--- (7)} \]

\[ L = 2\sqrt{\frac{d^2}{4} + \frac{4d^2}{3}} \text{ where } "d" \text{ is more than 5% of } "s" \text{ (approx)} \text{--- (8)} \]

\[ L = \frac{s^2}{8d} \left[ 4d \sqrt{1 + \frac{16d^2}{s^2}} + \log_e \left( \frac{4d}{s} \sqrt{1 + \frac{16d^2}{s^2}} \right) \right] \quad \text{--- (Exact)} \text{--- (9)} \]

\[ L = \frac{s}{2} \left[ \sqrt{1 + \frac{16d^2}{s^2}} + \frac{s^2}{4d} + \log_e \left( \frac{4d}{s} \sqrt{1 + \frac{16d^2}{s^2}} \right) \right] \quad \text{--- (9a)} \]

\[ L = \frac{p}{2d} \left[ 2d \sqrt{1 + \frac{4d^2}{p^2}} + \log_e \left( \frac{2d}{l} \sqrt{1 + \frac{4d^2}{l^2}} \right) \right] \quad \text{--- (9b)} \]

\[ L = \frac{p}{2d} \left[ \sqrt{\tan^2 \alpha \cdot \sec \alpha + \log_e (\tan \alpha + \sec \alpha)} \right] \quad \text{--- (9c)} \]

Based on the exact formula (9), the "Span Factors" below will be of use:

\[ \begin{array}{|c|c|c|c|c|}
\hline
s/d & Fraction & Span Factor & Tan. & Sec. \\
\hline
20 & 1/5 & 1.098042 & .8 & 1.2800 \\
15 & 3/20 & 1.057039 & .6 & 1.1662 \\
12-1/2 & 1/8 & 1.040188 & .5 & 1.11803 \\
11 & 1/9.09 & 1.031239 & .4 & 1.0925 \\
10 & 1/10 & 1.026055 & .4 & 1.0770 \\
9.09 & 1/11 & 1.021614 & .3636 & 1.0641 \\
8.33 & 1/12 & 1.018218 & .3333 & 1.0541 \\
8 & 1/12.5 & 1.016867 & .32 & 1.0500 \\
7.7 & 1/13 & 1.015600 & .3077 & 1.0462 \\
7-1/7 & 1/14 & 1.014350 & .2857 & 1.0400 \\
6-2/3 & 1/15 & 1.011722 & .2667 & 1.0349 \\
6-1/4 & 1/16 & 1.010316 & .25 & 1.0308 \\
5.88 & 1/17 & 1.009139 & .2353 & 1.0273 \\
5.55 & 1/18 & 1.008154 & .2222 & 1.0244 \\
5-1/4 & 1/19 & 1.007323 & .2105 & 1.0219 \\
5 & 1/20 & 1.006626 & .2 & 1.0198 \\
\hline
\end{array} \]

**Fixed Ends** Inclined Span

\[ H = \frac{wm^2}{2d_m} \quad \text{--- (12)} \]

\[ \tan z = \frac{2d_m}{m} \quad \text{--- (13)} \]

\[ \sec z = \sqrt{1 + \frac{4d^2}{m^2}} \quad \text{--- (14)} \]

\[ \tan \beta = \frac{2(D - d_m)}{m} \quad \text{--- (15)} \]

\[ \sec \beta = \sqrt{1 + \tan^2 \beta} \quad \text{--- (16)} \]

\[ T = H \cdot \sec z, \text{ at high support} \quad \text{--- (17)} \]

\[ T = H \cdot \sec \beta, \text{ at low support} \quad \text{--- (18)} \]

Equation of curve, origin at left support, xx axis thru origin:

\[ y = \frac{4dx}{s^2} (s-x) + x \cdot \tan \theta \quad \text{--- (19)} \]

Lowest point \( z \), where \[ m = \frac{s}{2} \left( 1 + \frac{s}{4d} \cdot \tan \theta \right) \quad \text{--- (20)} \]
Approximate formulas for cableways on inclined spans:
\[ H = \frac{ws^2}{8d} \]  
(21)

\[ T = H \cdot \sec \alpha = \frac{ws^2}{8d \cdot \cos \alpha} \]  
(22)

\[ d_a = \frac{4x_d}{s}(s - x) \]  
(23)

\[ d_m = \frac{4amn}{s^2} \cdot \frac{wnn}{2H} \]  
(24)

\[ m = \frac{s(4d + D)}{8d} \]  
(25)

Concentrated Loads on Suspended Spans. (Fixed Ends) Horizontal Span.

\[ H = \frac{(wl + P)l}{2d} \]  
(26)

or \[ H = \frac{ws^2 \cdot Ps}{8d \cdot 4d} \]  
(26a)

Load “P” lbs. Cable at “w” lbs. per ft.

\[ d_a = \frac{(P + ws)(s - x)}{2H} \]  
(27)

\[ d_a = \frac{2d + 8d_{a}}{s} \]  
(28)

\[ \tan \alpha = \frac{2d + 8d_{a}}{s} \]  
(29)

Approx. cable length: \[ L = 2 \left\{ \sqrt{\frac{s^2}{a} + \frac{8d_{a}^2}{9}} + \frac{\sqrt{2d^2 + d^4}}{3} \right\} \]  
(30)

Another more accurate method of figuring “d_a” is from:
\[ d_a = d \pm \triangle d \]  
(38)

\[ \triangle d = \frac{15 \triangle L}{16(5r^2 - 24r)} \]  
(39)

where \( r = d/s \)

\[ \triangle L = \frac{H_s}{AE} \left( 1 + \frac{16d^2}{3} \right) \]  
(40)

\[ \triangle L = H_sL \]  
(40a)

\[ H_1 = \frac{Ps}{4d} \]  
(41)

(for Concentrated Load “P”);

or \[ H_1 = \frac{ps^2}{8d} \]  
(42)

(For a uniform superimposed load “p” lbs. per foot over entire span);

or \[ H = \frac{ps^2}{8d}(1 - 4k^2) \]  
(43)

(For a partially loaded section of “p” lbs. per foot centrally located, similar to Figure 15, Page 299.)

Suspended length, unloaded (dotted curve),

\[ L_0 = s + \frac{8d_a^2}{3s} \]  
(44)

Elongation under stress of \( T_A = \triangle t_A = \frac{T_A L_0}{AE} \)  
(45)

where “A” is metallic area of rope, square inches; and \( E_r \) is Modulus of Elasticity of rope, Section 8, page 11.

Length on Ground, \( L_g = L_0 - \triangle \)  
(46)

Further correction for temperature from Equation 86.

\[ d = \frac{(ws + 2P)mn}{2H \left[ \frac{ws^2 + 4P}{mn} \right]} \]  
(47)

\[ \tan \alpha = \frac{2d + 8d_a}{s} \]  
(48)

\[ \tan \beta = \frac{2d - D + 8d_a}{s} \]  
(49)

\[ T_a = \frac{T_a}{\sin (\alpha \pm \beta) + \frac{ws^2}{8d \cos s}} \]  
(50)
\[ T_1 = \frac{P \cos \alpha}{\sin(\alpha \pm \beta)} + \frac{w s^2}{8d \cos \theta} \quad (51) \]
\[ T_2 = \frac{P \cos \beta}{\sin(\alpha \pm \beta)} + \frac{um}{2d \cos \theta} \quad (52) \]
\[ T_3 = \frac{P \cos \alpha}{\sin(\alpha \pm \beta)} + \frac{um}{2d \cos \theta} \quad (53) \]

"\( \beta \)" is + or - as vertex is below or above the lower support.

The suspended length of rope for a single load "\( P \)" at any position, "\( m \)" from left support and with a difference in elevation of supports of "\( D \)". Fig. VII, as given by F. C. Carstaphen is,
\[ L = s + \frac{P m}{2T} + \frac{w s^2}{24T^3} + \frac{D^2}{2s} \quad (54) \]
which for "\( P \)" at midspan, or where \( m = \frac{s}{2} \)
\[ L_m = s + \frac{P}{4T} + \frac{w s^2}{24T^3} + \frac{D^2}{2s} \quad (54a) \]
and the length with empty span ("\( P \)" removed):
\[ L_{m0} = s + \frac{w s^2}{24T^3} + \frac{D^2}{2s} \quad (54b) \]

**"Aerial Tramways" by F. C. Carstaphen presented before A.S.C.E. November 3, 1937.**

**Counterweighted ropes**

(Constant Tension)

The rope supporting a load does not present a continuous curve, but consists of two parabolas. The curve of the path of a moving load is continuous and may be expressed by:
\[ y = \frac{w s}{2T} + \frac{P_{mn}}{sT} + w s \tan \theta \quad (55) \]

If a point on the curve to the left of "\( P \)" is considered, the equation of the left parabola is:
\[ y_1 = \frac{w s}{2T} + \frac{P_{zn}}{sT} + x_1 \tan \theta \quad (56) \]
and for the right parabola:
\[ y_4 = \frac{w s}{2T} + \frac{P_{zn}}{sT} + x_2 \tan \theta \quad (57) \]

with "\( P \)" at midspan, \( m = n = \frac{s}{2} \) then,
\[ y = \frac{w s^2}{8T} + \frac{D}{2} \quad (58) \]

If the supports are the same elevation, the last term in equations 55-58 becomes zero.

The following equations are derived from Eqs. 55-58:
\[ d_x = \frac{w s^2}{8T \cos \alpha} \quad (59) \]
\[ d_y = \frac{w x z + P z}{2T \cos \alpha} \quad (60) \]
\[ d_z = \frac{w s^2 + 2P s}{8T \cos \alpha} \quad (61) \]
\[ d_m = \frac{mn(2s + 2P)}{2s T \cos \alpha} \quad (62) \]

In the formulas, 55-65, on preceding pages, it is not always easy to obtain the value of \( H \) or \( \cos \alpha \), \( (H = T \cdot \cos \alpha) \) so that the value of "\( T \)" is used. This substitution of "\( T \)" for "\( H \)" would reduce "\( d \)" by a maximum of about 5%.
Cable spans may be divided into two general classes, Anchored Spans, and Counterweighted Spans. In each of these divisions, we find it necessary to solve for stresses and deflections of uniformly loaded spans and also of spans supporting one or more individual concentrated loads. It is, therefore, necessary to analyze the conditions of each problem carefully and the following points must be considered:

1. Horizontal distance between supports.
2. Difference in elevation between supports.
3. Maximum allowable deflection, measured vertically from chord to cable.
4. Length of cable between supports.
5. Weight per foot of cable, to which must be added in certain cases the additional weight imposed by snow and ice.
6. Maximum load to be supported by the cable.
   a. Load uniformly distributed over the length of the span.
   b. A single load supported at any point in the span.
   c. Multiple individual loads.
7. Is the cable anchored at both ends or is it anchored at one end and counterweighted at the other end?
8. Modulus of elasticity in tension.
9. Wind loads on the cable and on the suspended load.
10. Changes in length of cable due to changes in temperature.

Since our purpose is to present means for obtaining results quickly, we will not give derivations of the following formulas. Computations are simplified by the assumption that uniform loading is distributed horizontally, and that the cable assumes a parabolic arc. For the great majority of cases encountered in practice, the results thus obtained are sufficiently accurate. If special cases occur where the ratio of deflection to span is very large, then the catenary equations should be applied. These are available in several textbooks.

The following nomenclature will be used:

\[ A = \text{Net cross sectional area of cable.} \]
\[ a = \text{Horizontal spacing of loads.} \]
\[ b = \frac{n(n-1)}{2} \]
\[ c = \frac{u(u-1)}{2} \]
\[ e = \text{Base of Naperian system of logarithms} \approx 2.7182818. \]
\[ E = \text{Modulus of elasticity in tension.} \]
\[ G = \text{Weight of an individual concentrated load.} \]
\[ h = \text{Vertical difference in elevation of supports.} \]
\[ k = \text{Ratio of deflection to span} = \frac{Y}{s} \text{ for level spans and} \frac{wscos^{2}a}{8t} \text{ for inclined spans.} \]
\[ L_1 = \text{Length along cable when the cable only is supported in a span.} \]
\[ L_2 = \text{Hypothetical length along cable at zero tension.} \]
\[ l = \text{Length along cable when either a uniformly distributed load or one or more concentrated loads are suspended.} \]
\[ m = \text{Horizontal distance from left support to the first load.} \]
n = Number of concentrated loads.
P = Change in total length of cable per pound of tension = \frac{L}{AE}

s = Horizontal distance between supports.
s_1 = Chord length of sub-span between load and support or between two loads.
t = Horizontal component of cable tension.
t' = Maximum cable tension at left support.
t'' = Maximum cable tension at right support.
t_e = Erection tension of empty cable in an anchored span.
u = Number of loads to left of xy in a multiple loaded span.
w = Weight per foot of horizontal length of span for a uniformly distributed load, \(w = \frac{w'}{\sec \alpha}\).
w' = Weight per foot of uniformly distributed load along the cable, which is assumed for purposes of parabolic curve calculations, as equivalent to uniformly distributed load along the chord.
w'' = Weight per foot of uniformly distributed load along the cable for purposes of catenary curve calculations.
x = Horizontal distance from support to xy.
y = Vertical deflection from support to xy.
y_e = Vertical deflection from support at center of span.
z = A term in the general formula for multiple loaded counterweighted spans.
\(\alpha\) = Alpha = Angle between the horizontal and a chord between supports.
\(\beta_1\) = Beta_1 = Angle between the horizontal and a tangent to a cable curve at the left support.
\(\beta_2\) = Beta_2 = Angle between the horizontal and a tangent to a cable curve at the right support.
\(\beta_3\) = Beta_3 = Angle between the horizontal and a tangent to a cable curve at any point in a span.
\(\beta_4\) = Beta_4 = Angle between the horizontal and a tangent to a cable curve at a load.
\(\lambda\) = Lambda = Change in length of cable per foot of length, per pound of tension.
\(\Delta\) = Delta = Total change in length of cable = \(\lambda \pm L_2\).

\(\theta\) = Theta = Angle between the horizontal and the chord of a half span.

\(\sec\) = Secant of an angle = \(\frac{1}{\cos}\)

ANCHORED SPANS are principally employed for supporting electrical cables, for guy lines, for suspension bridges, and usually for track cables of cableways and reversible aerial tramways where a single moving load is supported in a clear span.

When a cable span is erected, anchored at both ends, and a load of any kind supported from the cable, the deflection increases because of the elastic properties of the cable. The tension also increases when the load is applied.

It is necessary to select the size, construction, and grade of the cable, with a proper factor of safety, after having determined the maximum tension in the cable due to dead and live loads. It is then necessary to erect the cable at such a deflection that the maximum safe working tension will not be exceeded when the load is applied.

In the case of cableways with high self-supporting towers, the cable tension and deflection may be affected by yielding of the supports. A complete study of such a span includes the application of the theory of deflection in framed structures, but such a special condition does not come within the scope of this handbook. In all cases we will assume that cables are anchored to rigid supports or immovable ground anchorages.

The determination of the proper erection deflection and tension involves the use of the modulus of elasticity in tension for the particular construction of cable which is being used.

It is well known that the modulus of elasticity ranges between 28,000,000 and 30,000,000 for structural steel, but the modulus of elasticity of a wire cable, considering the cable as a whole, has various values depending on its construction, and also on the work that has been put into it.

The modulus can be appreciably increased by a prestressing operation. This is frequently done to bridge cables. In the case of track cables carrying rolling loads somewhat the same effect is secured after a period of operation, as most of the structural stretch is removed. See Modulf of Elasticity page 167, and Prestressed Strands and Ropes page 16.
When the tension is known, the center deflection is found from:

\[ \gamma_c = \frac{w s^2}{4t} \]  \hspace{1cm} (1)

and the deflection at any point in the span is:

\[ \gamma = \frac{w x(s-x)}{2t} \]  \hspace{1cm} (2)

When the center deflection is known, the horizontal component of tension is found from:

\[ t = \frac{w s^2}{8 \gamma_c} \]  \hspace{1cm} (3)

When the deflection at some point other than the center of span is known:

\[ t = \frac{w x(s-x)}{2 \gamma} \]  \hspace{1cm} (4)

\[ t' = t \sec \beta \]  \hspace{1cm} (5)

The cable slope at any point in the span is:

\[ \tan \beta = \frac{w}{t} \left( \frac{s}{2} - x \right) \]  \hspace{1cm} (6)

At either support the cable slope is:

\[ \tan \beta_1 \text{ or } \beta_2 = \frac{4 \gamma_c}{s} \]  \hspace{1cm} (7)

also \[ \tan \beta_1 \text{ or } \beta_2 = \frac{w s}{2t} \]  \hspace{1cm} (8)

When the tension is known, the length of cable is:

\[ L_1 \text{ or } L = s + \frac{w s^3}{24 t^2} \text{(approx.)} \]  \hspace{1cm} (9)

When the deflection is known: \( L_1 \text{ or } L \approx s \left[ \frac{1}{2} \sqrt{1 + 16k^2} + \frac{1}{8k} \log_2 \left( \frac{4k + \sqrt{1 + 16k^2}}{2k} \right) \right] \]  \hspace{1cm} (10)

An easier formula, giving closely approximate results is:

\[ L_1 \text{ or } L = s \left( 1 + \frac{8}{3} k^2 - \frac{32}{5} k^4 + \frac{256}{7} k^6 \right) \]  \hspace{1cm} (11)

Sufficient accuracy can be secured, for many of the cases encountered in practice, by contracting formula (11) to:

\[ L_1 \text{ or } L = s \left( 1 + \frac{8}{3} k^2 \right) \]  \hspace{1cm} (12)

In determining the erection tension for a uniformly loaded span, the values of \( L_1 \text{ and } t \), must satisfy the equation:

\[ L - L_1 = \frac{(t - t_2)}{AE} = P (t - t_2) \]  \hspace{1cm} (13)

By substitution of (9) for \( L_1 \) in (13) and using corresponding values of \( w \) and \( t_2 \),

\[ P t + L - (P t + s) = \frac{w s^3}{24 t^2} \]  \hspace{1cm} (14)

This equation can be solved for \( t_2 \), using the trial and error method.

EXAMPLE:

A 750,000 C.M. bare, hard drawn, stranded copper cable is to be supported across a river. Supports will be at the same elevation and 1350 feet apart. The copper cable is .9981 inches in diameter, and weighs 2,325 pounds per foot. Conditions require consideration of a coating of ice \( \frac{1}{4} \)" thick on both conductor and messenger, plus a horizontal wind load of eight pounds per square foot on the projected area of the ice coated cables. We are limited to a maximum center deflection of 75 feet.

(a) What are the specifications of the necessary messenger cable, assuming the same ice and wind loads?

(b) What is the cable slope at supports and at the quarter points of the span?

(c) What is the erection tension and deflection for the messenger strand only, assuming there are no ice or wind conditions at time of erection?

It is necessary to assume the diameter of messenger strand to figure the loading on the span. It may then be necessary to revise the figures if the first selection does not prove suitable. We will assume a \( \frac{1}{4} \)" diameter strand weighing 1.581 pounds per foot.

Copper cable + ice = 3.240 pounds per foot
Messenger strand + ice = 2.421 pounds per foot
Total vertical load = 5.661 pounds per foot
Horizontal wind load on both cables = 2.582 pounds per foot
Total resultant load per foot = 6.222 pounds

Then from (3) \[ t = \frac{6.222 \times 1350^2}{8 \times 75} = 18,900 \text{ pounds} \]

Then from (8) \[ \tan \beta_1 = \frac{6.222 \times 1350}{2 \times 18900} = .2222, \beta_1 = 12.32^\circ \]

Then from (5) \[ t' = 18900 \times \sec 12.32^\circ = 19365 \text{ pounds} \]

Then from (6) when \( x = 337.5 \text{ feet} \)

\[ \tan \beta_2 = \frac{6.222}{18900} (675 - 337.5) = .1111, \beta_2 = 6.20^\circ \]

With a factor of safety of 4, the required breaking strength will be \( 4 \times 19365 = 77460 \text{ pounds} \). Page 117 shows \( \frac{1}{4} \)" diameter, 19 wire, Extra Galvanized Extra High Strength Strand has a breaking strength of 79,700 pounds, and will be satisfactory for the purpose intended. From Page 117 we find \( W \) to be 1,581 pounds per foot. From Page 166 we find \( A \) to be .4444 square inches.

(9) \[ L = 1350 + \frac{6.222 \times 1350^2}{24 \times 18900} = 1361.110 \text{ ft} \]

In order to set up (14) in convenient form, first calculate the following:

\[ P = \frac{1361.110}{.4444 \times 21,000,000} = .0001458 \text{ feet per pound} \]

\[ P t + s = (.0001458 \times 18900) + 1350 = 1352.756 \text{ ft} \]

\[ \frac{w s^3}{24} = \frac{1.581^2 \times 1350^3}{24} = 256,240,000 \]

Substituting these values in (14),

\[ .0001458 t_2 + 8.354 = \frac{256,240,000}{t_2} \]

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The following shows the results of a series of tentative computations for assumed values of \( t_c \) until the above equation is satisfied (the values in the last two columns are equal).

\[
\frac{t_c}{.0001458 t_c} + 8.354 \frac{256,240,000}{t_c^2} = 9.098 \quad 9.852
\]

\[
5100 \quad .744 \quad 9.098 \quad 9.852
5200 \quad .758 \quad 9.112 \quad 9.476
5290 \quad .771 \quad 9.125 \quad 9.157
5299 \quad .773 \quad 9.127 \quad 9.126
\]

\( t_c = 5299 \) pounds.

From (1): \( y_c = \frac{1.581 \times 1350^2}{8 \times 5299} = 67.97 \) ft.

From (9): \( l_1 = 1350 + \frac{1.581^2 \times 1350}{24 \times 5299^2} = 1359.126 \) ft.

Therefore:

(a) One piece \( \frac{3}{8} \)" diam, 19 wire Extra Galvanized Extra High Strength Strand with sockets attached so as to give a length of 1359.13 feet center to center of supports.

(b) Maximum cable slope at supports = 12°-32'.
   Maximum cable slope at quarter points of span = 6°-20'.

(c) Erection tension = 5299 pounds
   Erection deflection = 67.97 ft.

### LENGTH AND MAXIMUM TENSION

The following table gives factors for obtaining maximum tension \( T \) at the supports of a uniformly loaded level span when \( w \), the weight per horizontal foot and \( s \), the horizontal length of span, are known. See column 2. The close relation between the parabola and the catenary is shown by a comparison of the values in columns 2 and 3. Column 3 gives the factor for obtaining \( T' \) when \( w' \), the weight per foot along the cable, and \( s \) is known. The length of a uniformly loaded level span, based on a parabolic curve, can be obtained from the factors in column 4. If the span is inclined see formulas (24) and (25).

The factors in column 4 can also be used for the catenary for \( k \) ratios up to 0.12 with an error less than 0.02%, and for \( k \) ratios as high as 0.20 with an error of only 0.1%.

<table>
<thead>
<tr>
<th>CABLE LENGTH AND MAXIMUM TENSION</th>
<th>FACTORS FOR MAXIMUM TENSION</th>
<th>CABLE LENGTH FACTORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of center deflection to chord length of span ( k )</td>
<td>When weight of load per foot of span, ( w ), is known ( T' = w x \times \text{factor} )</td>
<td>When weight of load per foot of cable, ( w' ), is known ( T' = w' x \times \text{factor} )</td>
</tr>
<tr>
<td>COLUMNS 1</td>
<td>COLUMNS 2</td>
<td>COLUMNS 3</td>
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<tr>
<td>.01</td>
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## Cable Length and Maximum Tension (Cont.)

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<th>Ratio of center deflection to chord length of span k</th>
<th>Factors for Maximum Tension</th>
<th>Cable Length Factors</th>
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<tbody>
<tr>
<td></td>
<td>When weight of load per foot of span, ( w_x ), is known ( t' = w_x \times \text{factor} )</td>
<td>To get length of cable, multiply total span by factor below ( L_3 = s \times \text{factor} )</td>
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[Image of a large cable structure]
The following formulas give the increments of deflection and slope due to inclination of the chord. "Down" slopes are usually considered as plus values and "up" slopes as minus values.

\[ y_e = \frac{ws^2}{8t} + \frac{h}{2} \]

\[ \tan \alpha = \frac{h}{s} \]

At any point—

\[ y = \frac{wx(s-x)}{2t} \pm x \tan \alpha \]

\[ \tan \beta_1 = \frac{ws}{2t} + \tan \alpha \]

\[ \tan \beta_2 = \frac{ws}{2t} - \tan \alpha \]

\[ \tan \beta_3 \text{ (at any point)} = \frac{w}{t} \left( \frac{s}{2} - x \right) \pm \tan \alpha \]  

When center deflection is known:

\[ t = \frac{ws^2}{8y_e - 4h} \]  

(21)

Low point of an inclined span occurs when \( \tan \beta_3 = 0 \)

\[ \therefore x = \frac{s}{2} + \frac{t}{w} \tan \alpha \]  

(15)

When deflection at any other point is known:

\[ t = \frac{w(x-s)}{2(y-x \tan \alpha)} \]  

(16)

\[ t' = t \sec \beta_1 \]  

(22)

\[ t'' = t \sec \beta_2 \]  

(23)

To find the lengths of cable in an inclined span formulas (9) and (11) are modified:

\[ L_1 \text{ or } L = \sqrt{s^2 + h^2 + \frac{w^2 s^3 \cos^2 \alpha}{24t^2}} \text{ (approx.)} \]  

(18)

\[ L_1 \text{ or } L = \sqrt{s^2 + h^2 \left( 1 + \frac{8}{3} k^2 - \frac{32}{5} k^4 + \frac{256}{7} k^6 \right)} \]  

(24)

(25)

It will be seen that the solutions for inclined spans are quite similar to those for level spans.
LEVEL SPAN — SINGLE LOAD AT CENTER — ANCHORED

The deflection produced by a concentrated load suspended midway between two fixed points A and B forms two equal sub-chords AC and CB. The cable assumes two catenary arcs which intersect at C. The following formulas are, however, based on the parabola, as the difference in results is negligible.

The center deflection is found from:

\[ y_c = \frac{Gs}{4t} + \frac{ws^2}{8t} = \frac{s(2G + ws)}{8t} \]  \hspace{1cm} (26)

and \[ t = \frac{s(2G + ws)}{8y_c} \]  \hspace{1cm} (27)

\[ t' = t \sec \beta_1 = t \sec \beta_2 = t'' \]  \hspace{1cm} (28)

\[ \tan \beta_1 = \frac{G + ws}{2t} \]  \hspace{1cm} (29)

Example: A rolling load weighing 2000 pounds is to be supported in a level span 2000 ft. long by a cable anchored at both ends. The deflection must not exceed 83 feet. No wind or ice conditions.

(a) What are the specifications of the cable?
(b) What is the maximum tension in the cable?
(c) What is the slope of the supports with the load at center of span?
(d) What is the cable length between supports, with no rolling load on the cable?
(e) What is the erection tension and erection deflection of the cable?

It is necessary to assume a size and grade of cable for the calculations. If the first selection does not prove suitable, the calculations must be revised. We shall assume that a 1¼" diameter Standard Grade Locked Coil Cable will be suitable.

Since this is a level span, \( \alpha = 0 \) and \( w = w' \)

\[ w = 3.16 \text{ pounds per foot} \]  \hspace{1cm} (from page 111). \( A = 0.8567 \text{ square inches} \) (from page 166).

From (27):

\[ t = \frac{2000(2 \times 2000 + 3.16 \times 2000)}{8 \times 83} = 31,084 \text{ pounds} \]

From (29):

\[ \tan \beta_1 = \frac{2000 + (3.16 \times 2000)}{2 \times 31084} = 0.1338 \]  \hspace{1cm} (1¼"")

From (28):

\[ t' = 31084 \times 1.0089 = 31360 \text{ pounds} \]

The maximum cable length occurs when load is at center of span.

\[ s_1 = \sqrt{\left(\frac{s}{2}\right)^2 + y_c^2} = \sqrt{1000^2 + 83^2} = 1003.439 \text{ ft.} \]

\[ \tan \theta = \frac{83}{1000} = 0.083 \]  \hspace{1cm} (4°.45)

\[ L = 2 \left( s_1 + \frac{w' \left(2 \times \frac{1}{2} \cos^2 \theta\right)}{24 \times t'^2} \right) \]

\[ = 2 \left( 1003.439 + \frac{3.16^2 \times 1000 \times \cos^2 4°.45}{24 \times 31084^2} \right) \]

\[ = 2007.730 \]

In order to set up (14) in convenient form, first calculate the following:

\[ P = \frac{2007.730 \times 0.8567 \times 19,000,000}{24} = 0.001233 \]

\[ Pt = 0.001233 \times 31,084 + 2000 = 2003.833 \]

\[ \frac{w'_t}{24} = \frac{3.16^2 \times 2000^2}{24} = 3,328,533,333 \]

Substituting these values in (14):

\[ \frac{\tan \beta_1}{2} = \frac{0.001233}{2} = 3,328,533,333 \]

The following shows the results of a series of slide rule computations for assumed values of \( t_c \) until the above equation is satisfied (the values in the last two columns are equal).

\[ t_c \quad 0.001233t_c \quad 3.897 \quad 3,328,533,333 \]

\[ 22,000 \quad 2.713 \quad 6.610 \quad 6.877 \]

\[ 22,300 \quad 2.750 \quad 6.647 \quad 6.993 \]

\[ 22,360 \quad 2.757 \quad 6.654 \quad 6.657 \]

\[ t_c = 22,360 \text{ pounds} \]

from (1):

\[ y_c = \frac{3.16 \times 2000^2}{8 \times 22360} = 70.66 \text{ ft.} \]

from (9):

\[ L = 2000 + \frac{3.16^2 \times 2000^2}{24 \times 22360^2} = 2006.66 \text{ ft.} \]

(a) With a factor of safety of 3.2, the required breaking strength will be 3.2 \times 31360 = 100352 \text{ pounds} = 50.18 \text{ tons}. The breaking strength of a 1¼" diameter Standard Grade Locked Coil Cable is 54 tons (from page 111). Therefore, this size cable is satisfactory, and our Locked Coil Cable is the most suitable construction where rolling loads are to be handled. If the proposed installation is temporary, or if first cost of the cable is a prime consider-
Formula (32) can be applied to inclined spans by adding $\frac{hx}{s}$, which becomes $\frac{h}{2}$ when $x = \frac{s}{2}$. Then, for inclined spans:

$$y = \frac{x(ws^2 + 2G)(s-x)}{2t(ws^2 + 4G \sqrt{x(s-x)})} + \frac{hx}{s}$$ (34)

**MULTIPLE LOADS IN ANCHORED SPANS**

Multiple loads in anchored spans are seldom encountered in practice. However, the subject is important enough to merit some attention. When speaking of multiple loads, it will be assumed loads are equal in amount and spaced uniformly.

The loads should be placed symmetrically about the center line of the span to compute the maximum tension or deflection in the span. Use formula (52), page 188, to determine the deflection and formula (34) to determine the maximum tension. To determine the length along the cable at maximum tension, consider the loads as stationary in the position stated above and treat the lengths of cable between supports and the first load, and the lengths between loads, as separate spans. After this length, $l$, has been determined, the erection tension, deflection of empty cable, etc., are calculated by the trial method in a similar manner to that for a single load in an anchored span.
LEVEL SPAN—UNIFORMLY LOADED—COUNTERWEIGHTED

The tension and deflection of either an anchored or a counterweighted span are the same, under the same conditions of loading, when the cable supports a uniformly distributed load. However, an important difference occurs when the live load is removed. In the case of an anchored span, the deflection and length of the cable remain constant, except as they are affected by the elastic properties of the cable, backstays, and supports. The tension, however, decreases when the live load is removed. Comparing this performance with a counterweighted span, we find that the tension remains constant when the live load is removed, while the deflection and length of the cable decrease in proportion to the change in loading. These are the effects due to equalizing the moment-sum of all forces for any origin of moments.

The same comparison holds true of spans supporting one or more individual concentrated loads, when the loads are so placed as to produce the maximum deflection.

The use of a counterweighted track cable for rolling loads results in a constant angle under the load, the angle whose tangent is \( \frac{G}{T} \), at all points of a span. Also, it produces a smaller angle at each support than would be the case with an anchored span. These two factors are of definite advantage in the design of aerial tramways having intermediate supports.

Apply formulas (1) to (12) inclusive, page 178, under “Anchored Spans.”

INCLINED SPAN—UNIFORMLY LOADED—COUNTERWEIGHTED

Apply formulas (15) to (25) inclusive, page 181, under “Anchored Spans.”
LEVELSPAN—SINGLELOAD AT CENTER—COUNTERWEIGHTED

In a constant tension span the deflection at the load may be determined from:

\[ y = \frac{Gx(s-x)}{st} + \frac{wx(s-x)}{2t} \]  

(35)

Also the deflection of the cable may be determined for any point in the span, with the load at any point, \( x_1 \), \( y_1 \) being coordinates to points to the left of \( G \) and \( x_2 \), \( y_2 \) being coordinates of points to the right of \( G \).

\[ y_1 \text{ (points left of } G) = \frac{Gx_1}{st} (s-m) + \frac{wx_1}{2t} (s-x_1) \]  

(36)

\[ y_2 \text{ (points right of } G) = \frac{Gm}{st} (s-x_2) + \frac{wx_2}{2t} (s-x_2) \]  

(37)

The cable slope at left support, when \( x_1 = 0 \), is:

\[ \tan \beta_1 = \frac{G}{st} (s-m) + \frac{w}{2t} \]  

(38)

The cable slope at right support, when \( x_2 = s \), is:

\[ \tan \beta_2 = \frac{Gm}{st} + \frac{ws}{2t} \]  

(39)

The cable slope at any point between the load and either support is:

\[ \tan \beta_0 \text{ (points to left of } G) = \frac{G}{st} (s-m) + \frac{w}{t} \left( \frac{s}{2} - x_1 \right) \]  

(40)

\[ \tan \beta_3 \text{ (points to right of } G) = \frac{Gm}{st} + \frac{w}{t} \left( x_2 - \frac{s}{2} \right) \]  

(41)

When \( x = m \), the slope at and to the left of the load is:

\[ \tan \beta_1 \text{ (sloping to left of } G) = \frac{G}{st} - \frac{Gx}{st} + \frac{w}{t} \left( \frac{s}{2} - x \right) \]  

(42)

The slope at and to the right of the load is:

\[ \tan \beta_3 \text{ (sloping to the right of } G) = \frac{Gx}{st} + \frac{w}{t} \left( x - \frac{s}{2} \right) \]  

(43)

The tangent of the angle under the load is equal to

\[ (42) + (43) = \frac{G}{t} \]

If we take half the difference between the angles obtained from (42) and (43), the tangent of the resulting angle will be the slope which a moving load must climb. The maximum slope thus obtained will determine the maximum pull on a hauling rope.
INCLINED SPAN—SINGLE LOAD AT ANY POINT—COUNTERWEIGHTED

In these formulas, as in all others, we have placed the higher support at the left-hand end of span, and have made this point the origin of moments.

For \( y \) — at the load — add to formula (35)
\[
x \tan \alpha
\]
(44)

For \( y_1 \) — points left of \( G \) — add to formula (36)
\[
x_1 \tan \alpha
\]
(45)

For \( y_2 \) — points right of \( G \) — add to formula (37)
\[
x_2 \tan \alpha
\]
(46)

The cable slopes are determined by taking the chord into account as an additional term in the above equations.

\[
\tan \beta_1 = \text{at left support — formula (38)}
\]
(47)
\[
+ \tan \alpha
\]

\[
\tan \beta_2 = \text{at right support — formula (39)}
\]
(48)
\[
- \tan \alpha
\]

\[
\tan \beta_3 = \text{points to left of \( G \) — formula (40)}
\]
(49)
\[
\pm \tan \alpha
\]

\[
\tan \beta_4 = \text{points to right of \( G \) — formula (41)}
\]
(50)
\[
\pm \tan \alpha
\]

\[
\tan \beta_5 = \text{at the load — formula (42) and (43)}
\]
(51)

EXAMPLE: A 2,000 pound rolling load is to be supported on an inclined span 800 ft. long with difference in elevation of 67 ft. The cable is \( 1 \frac{3}{4} \)" diameter Standard Grade Locked Coil; \( w' = 4.73 \) pounds per foot (from Page 111). \( A = 1.280 \) sq. in. (from Page 166). The center deflection must not exceed 18 ft. from the chord.

(a) What is the horizontal component of cable tension with load at center of span?

(b) What is the slope of the cable at the higher support

(1) with the load at center of span, (2) with the load 100 ft. horizontally away from the upper support and (3) with the cable unloaded?

(c) What is the center deflection of the unloaded cable?

\[
\tan \alpha = \frac{67}{800} = 0.08375, \alpha = 4^\circ - 47', \sec \alpha = 1.0035
\]

\[
w = 4.73 \times 1.0035 = 4.75
\]

From (31), \( t = \frac{800 (2 \times 2000 + 4.75 \times 800)}{8 \times 51.5 - 4 \times 67} \)

\[
= 43,333 \text{ pounds}
\]

From (47), \( \tan \beta_1 = \frac{2000 \times 700 + 4.75 \times 800}{800 \times 43333 + 2 \times 43333 + 800} \)

\[
= 0.1507, \beta_1 = 8^\circ - 34'\]

From (47), \( \tan \beta_2 = \frac{2000 \times 700 + 4.75 \times 800}{800 \times 43333 + 2 \times 43333 + 800} \)

\[
= 0.1507, \beta_1 = 9^\circ - 32'\]

From (47), \( \tan \beta_3 = \frac{4.75 \times 800}{2 \times 43333 + 800} \)

\[
= 0.1276, \beta_1 = 7^\circ - 16'\]

From (15), \( y_s = \frac{4.75 \times 800^2}{8 \times 43333} + \frac{67}{2} = 42.27 \text{ ft.} \)

(a) 43,333 pounds.

(b1) Slope 8°-34' with load at center of span.

(b2) Slope 9°-32' with \( m = 100 \) feet.

(b3) Slope 7°-16' with cable unloaded.

(c) Center deflection 42.27 feet with cable unloaded.
LEVEL SPAN — MULTIPLE LOADS — COUNTERWEIGHTED

A cable supporting multiple loads forms a series of parabolic arcs between the loads. For many cases encountered in practice, it will be sufficiently accurate to calculate spans carrying more than five loads as uniformly loaded spans. If this is done, the load per foot equals weight of cable plus $G/a$.

However, the general formula for deflection $y$, at any point $xy$, of a span supporting $n$ loads of uniform spacing and equal weight, the cable tension being constant, is:

$$y = \frac{G}{t} \left[ \frac{t}{(n-u) - m} \left( \frac{x}{s} - u \right) - \frac{bx}{s} - c \right] + \frac{wx(s - x)}{2t}$$  \hspace{1cm} (52)

If $z = \left[ \frac{t}{(n-u) - m} \left( \frac{x}{s} - u \right) - \frac{bx}{s} - c \right]$

Then $y = \frac{2Gz + wx(s - x)}{2t}$  \hspace{1cm} (53)

and $t = \frac{2Gz + wx(s - x)}{2y}$  \hspace{1cm} (54)

The cable slope at any point may be found from the general formula:

$$\tan \beta_3 = \frac{G}{t} \left[ \frac{(n-u) - \frac{nm}{s} + \frac{ab}{s}}{r} \right] + \frac{w}{t} \left( \frac{s}{2} - x \right)$$  \hspace{1cm} (55)

Example: A 1 3/4" diameter Standard Grade Locked Coil Cable is to be used to support 5 loads, each weighing 2000 pounds, and spaced uniformly 400 feet apart. Length of span 2000 feet. Horizontal component of working tension $t = 45,964$ pounds. $w = w' = 5.63$ pounds (from Page 111).

(a) What is maximum deflection?
(b) What is the slope of the cable at a point 500 feet from the support?

Maximum deflection occurs with one load at center of span.

$$x = 1000 \text{ ft.}$$
$$m = 200 \text{ ft.}$$
$$u = 2$$
$$b = \frac{5 \times 4}{2} = 10$$
$$c = \frac{2 \times 1}{2} = 1$$

From (52) $y_c = \frac{2000}{45964} \left[ \frac{1000(5-2) - 200}{2000 - 2} - 400 \left( \frac{10 \times 1000}{2000} - 1 \right) \right] + \frac{5.63 \times 1000 \times 1000}{2 \times 45964} = 56.566 + 61.244 = 117.81$ feet

From (55) with $x = 500$ feet, $u = 1$

$$\tan \beta_3 = \frac{2000}{45964} \left[ (5-1) - \frac{5 \times 200 + 400 \times 10}{2000} \right] + \frac{5.63}{45964} (1000-500) = .0653 + .0612 = .1265, \beta_3 = 7°-13'$

(a) $= 117.81$ feet
(b) $= 7°-13'$

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Inclined Span — Multiple Loads — Counterweighted

\[ y = \text{add to formula (52) and (53)} \times \tan \alpha \]  
\[ \text{Formula (54) becomes } t = \frac{2Gz + wx(s-x)}{2(y-x \tan \alpha)} \]  
\[ \tan \beta_0 \text{ is found by completing formula (55) with } \pm \tan \alpha \]

Wind and Ice Loads

The change in length of cables due to change in temperature has not been taken into account in the examples given in this section. In counterweighted spans such a change in length results in a small movement of the counterweight, the tension and deflection remaining constant. However, in anchored spans the change in length due to temperature changes results in changes in cable tension, and frequently the effect of such changes must be carefully considered.

To find the change in length, multiply the length of the cable by the number of degrees (F.) variation in temperature and the product by the coefficient 0.00000689 for steel rope wire.

Wind loads on cylindrical surfaces, such as wire cables, are determined from maximum wind velocities. If \( P \) equals wind pressure in pounds per square foot of projected area and \( V \) = actual wind velocity in miles per hour, then \( P = 0.0025V^2 \). This gives 4.0 pounds per square foot for 40 miles per hour, 12.2 pounds per square foot for 70 miles per hour, and 20.2 pounds per square foot for 90 miles per hour.

Where exceptionally severe sleet conditions occur, the cables are assumed to be covered with a coating of ice \( \frac{3}{8} \) inch thick, or a diametric total of \( \frac{1}{2} \) inches of ice plus the diameter of cable. Where the sleet conditions are less severe, the ice coating is assumed to be \( \frac{1}{2} \) inch thick, or a total of 1 inch plus the diameter of cable. Then wind load is based on the total diameter of ice plus cable, and the resultant cable load is determined from the horizontal wind load and vertical load of cable and ice. The weight of ice is approximately 56 pounds per cubic foot, or 0.0324 pounds per cubic inch.
TRAMWAY AND CABLEWAY DATA

A tramway can be described as a means of aerial or overhead transportation, wherein the material transported is carried in cars or buckets which are both supported and transported by means of wire rope.

Continuous Tramways

The continuous tramway consists of a series of carriers traveling around an endless circuit which connects the two terminals of the tramway.

1. The Single Rope Fixed Clip Tramway—In this case a single rope supports and transports the carriers. Carriers are fixed to the rope and do not leave it at the terminals. Although this type of tramway is obsolete for many industrial purposes, it is used very effectively for the modern ski lift and chair lift.

2. The Bicable or Double Rope Tramway—The carriers are supported by a stationary track cable and transported by a separate hauling rope. They are connected to the hauling rope by detachable grips and are removed from the track cable for loading and unloading at the terminals. Many industrial tramways now in use are of this type.

3. The Modern Monocable or Single Rope Saddle Clip Tramway—The carriers are supported and transported by a single endless rope. They rest on the rope and specially-designed saddle clips prevent slippage along the rope. The carriers are removed at the terminals for loading and unloading. This type of tramway is extremely simple and rugged in design and has an exceptionally low operating cost. It is in use for many industrial operations throughout the world.

Reversible Tramways

In these tramways, which are usually of the bicable type, one or two carriers shuttle between the terminals.

1. The To-and-Fro or Single Reversible Tramway—As its name implies, this type of tramway involves the use of a single carrier. It is frequently used for the disposal of slate and other refuse in mining operations.

2. The Jigback or Double Reversible Tramway—This type involves the use of two carriers. It is sometimes used for waste disposal but more often for transportation between two fixed points, such as transportation of ore from mine to mill where relatively short distances are involved. This type of tramway is often employed on passenger tramways to gain access to mountain-top resorts.

Cableways

The cableway is a special type of tramway which hoists as well as transports its loads. It is usually of the to-and-fro type. It is very often used in construction work and is a common sight during the installation of most large dams where it is used for placing concrete.

TRAMWAY CABLE CALCULATIONS

On continuous tramways the weights of the carriers and their loads are usually divided by their spacing and the quotient added to the rope weight in order to obtain an equivalent uniform weight \( w \) per unit of length along the tramway. The cable tensions and sags can then be found by the common parabolic formulas, such as equations 1 to 8, page 146. In the case of tramways, however, \( w \) in these formulas should be replaced by \( w \sec \alpha \). The following equation is another useful tool for the tramway designer:

\[
T_2 = T_1 + \frac{wB}{T_1} \]  

where \( T_1 \) = Tension in rope at any point (1).

\( T_2 \) = Tension in rope at a higher point (2).

\( B \) = Vertical rise from (1) to (2).

\( w \) = Equivalent uniform weight of rope and loads.

Note: For a bicable track cable, \( w \) is the weight of the track cable alone. For the hauling rope of the bicable or the rope of a monocable, \( w \) is the uniform weight of rope and buckets taken together.

Reversible tramways require special treatment, as the loads are concentrated rather than uniform. The following are methods of solution for a few of the problems involved:
A Typical Loaded Span

\[ H = \frac{(w_s K + 2P)K}{8f} \]  
\[ f_U = f_L = \frac{w_s K}{8H} \]  
\[ \tan \phi_{IL} = \frac{B_L - 4fL}{K_L} \]  
\[ \tan \phi_{2U} = \frac{B_U + 4fU}{K_U} \]  
\[ V_{IL} = H \tan \phi_{IL} \]  
\[ T_{IL} = H \sec \phi_{IL} \]  
\[ V_{2U} = H \tan \phi_{2U} \]  
\[ T_{2U} = H \sec \phi_{2U} \]

Short-Cut Method of Computing Equations 29 and 36

For convenience equation 29 is broken down into two sections, one representing \( H_1 \) for the load and the other \( H_2 \) for the rope weight:

\[ H_1 = \frac{PK}{4f} \]  
\[ H_2 = \frac{wK^2}{8f} \]

Then equation 29 may be written:
\[ H = H_1 + (H_2 \times \text{multiplier}) \]

Also equation 36 may be written:
\[ T = H \times \text{multiplier} \]

On Charts 2a, 2b and 2c, values of \( H_1 \), \( H_2 \), and the multiplier are plotted for various conditions. On all charts, each curve represents a different value of sag ratio \( \frac{f}{K} \), stated as a percentage.

Example for Horizontal Spans

\[ K = 2,000 \text{ ft.} \]
\[ f = 60 \text{ ft.} = 3\% \]
Load = 10,000 lbs.
\[ w = 10.44 \text{ lbs./ft. for two } 1\frac{1}{2}'' \text{ diam.} \]
Locked Smooth Coil Tramway Track Strands.
\[ wK = 2,000 \times 10.44 = 20,880 \]
\[ \frac{B}{K} = 0 \]

From Chart 2a, \( H_1 = 83,000 \) (for load = 10,000)

\[ ^{a} \quad 2b, H_1 = 87,000 \] (for \( wK = 20,880 \))

\[ ^{a} \quad 2c, \text{ Multiplier} = 1.002 \left( \text{for} \frac{B}{K} = 0 \right) \]

\[ H_2 \times \text{Multiplier} = 87,174 \]
\[ H_1 = 83,000 \]
\[ H = 170,174 \]
\[ T = 170,174 \times 1.002 = 170,500 \]

Factor of Safety = \( \frac{\text{Track Cable Strength}}{T} \)
\[ = \frac{2 \times 255,000}{170,500} = 3.00 \]
Example for Inclined Spans

From Chart 2a, \( H_1 = 83,000 \) (for load = 10,000 lbs.)

\( \frac{A}{2b}, H_2 = 100,500 \)

\( \frac{A}{2c}, \text{Multiplier} = 1.048 \left( \frac{B}{K} = .25 \right) \)

\[ H_3 \times \text{Multiplier} = 100,500 \times 1.048 = 105,300 \]

\[ H_1 = 83,000 \]

\[ H = 188,300 \]

\[ T = 1.048 \times 188,300 = 197,330 \]

Factor of Safety = \( \frac{\text{Track Cable Strength}}{T} \)

\[ = \frac{2 \times 292,000}{197,330} = 2.96 \]

Note: The recommended Factor of Safety for Locked Smooth Coil Tramway Track Strand is 3.0 and for Smooth Coil Tramway Track Strand it is 4.0.

\[ K = 2,000 \text{ ft.} \]

\[ f = 60 \text{ ft.} = 3\% \]

Load = 10,000 lbs.

\[ w = 12.10 \text{ lbs./ft.} \text{ for two 1\%}^\circ \text{ diam.} \]

Locked Smooth Coil Tramway Track Strands.

\[ B = 500 \text{ ft.} \]

\[ \frac{B}{K} = .25 \]

\[ nK = 24,200 \]
Path of Load

The designer is frequently interested in the path of travel of an individual concentrated load as it crosses a single span. The method of determining this so-called "Path of Load" depends on whether the track cables are maintained at a constant tension by means of counterweights or whether they are solidly anchored and, therefore, subject to changes in tension as the load traverses the span.

Track Cables Counterweighted

In this case the path of load can be computed as a parabola for all practical purposes. The formulas to be used are:

\[ y = 4f \left( \frac{x}{K} \right)^2 \left( 1 - \frac{x}{K} \right) \]  \hspace{1cm} \text{(41)}

\[ \tan \phi = \frac{4f}{K} \left( 1 - \frac{2x}{K} \right) + \frac{B}{K} \]  \hspace{1cm} \text{(42)}

and the meanings of the terms of these formulas are shown in Figure 3. The sag from the chord to the cable at mid-span when the load is at mid-span is represented by \( f \). (See "A Typical Loaded Span," page 155.)

\[ \text{When } x = \begin{align*} 0 & \quad y = 0 \\ .1K \text{ and } .9K & \quad .36f \\ .2K \text{ and } .8K & \quad .64f \\ .3K \text{ and } .7K & \quad .84f \\ .4K \text{ and } .6K & \quad .96f \\ .5K & \quad f \end{align*} \]

Figure 3

Track Cables Anchored

1. Single Span Tramway or Cableway with Relatively Short Backstays to the Anchorages — The path of load may be found with sufficient accuracy by the use of equations 43 and 44 shown in Figure 4.

\[ P = \text{Total weight of moving load (carriage, car, payload and moving ropes.)} \]

\[ w = \text{Weight per foot of track cable.} \]

\[ H = \text{Horizontal component of track cable tension with load at mid-span.} \]

\[ f, K, B, P, w \text{ given as design conditions.} \]

\[ w_e = \text{Effective weight per horizontal foot of track cable (} w \sec \alpha \text{).} \]

Then:

\[ H = \frac{(w_e K + 2P)K}{8f} \]  \hspace{1cm} \text{(29)}

\[ y = \frac{f}{2}(1 - \frac{x}{K}) (w_e K + 2P)^2 + x \frac{B}{K} \]  \hspace{1cm} \text{(43)}

\[ \tan \phi = \frac{(w_e K + 2P)^2 (K - 2x)(w_e K^2 + 2P \sqrt{Kx - x^3})^2}{2H} \]  \hspace{1cm} \text{(44)}

Figure 4

2. Multiple Span Tramway — In this case the spans adjacent to the one in which the path of load is desired have the effect of partial counterweights; thus the solution is more complicated. It is somewhere between the results obtained by Figures 3 and 4 and there is no satisfactory simple approximate method for this case. However, any tramway problem can be solved with the aid of the parabolic or catenary formulas mentioned above (equations 1-14, 18-25, 26, 27) together with the method outlined for a suspension bridge free cable (see Figure 2, page 150).